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From analysis to learning: Tensor-based assessment of latent similarity

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In data analysis, tensor decompositions such as the Canonical Polyadic Decomposition (CPD) and Block Term Decomposition (BTD) may be used as basic tools. They allow one to break a single data set into interpretable components. In CPD, the terms are rank-1, while, in the more general BTD, they have low multilinear rank. Instrumental is the mildness of the conditions under which the tensor decompositions are unique (for instance, no orthogonality requirements, as in the QR-factorization or singular value decomposition of matrices). Countless applications have been reported in telecommunication, array processing, audio and image processing, chemometrics, psychometrics, astrophysics, biomedical signal processing, bio-informatics, ... The mildness of the conditions goes together with the possibility of computational issues: the lack of properties such as orthogonality makes that the terms in a decomposition can be arbitrarily close to each other, and hence numerically difficult to separate; in some cases the problem can even be ill-posed.

In this talk we take the step from data analysis to data comparison, and from the decomposition of a single tensor to the assessment of the similarity between components of different tensors. Assessing similarity is a key task in pattern recognition and machine learning. We will show that, also in the latter setting, tensors provide fundamentally new possibilities beyond matrix techniques. Moreover, under mild conditions, the assessment of similarity can be done by conventional linear algebra, i.e. the estimation of angles between subspaces, solving sets of linear equations in least-squares sense and matrix eigenvalue decomposition. The number of terms and their multilinear rank (in the case of BTD) can be found as well. The results will be illustrated with applications.

References

[1] I. Domanov and L. De Lathauwer, *From computation to comparison of tensor decompositions*, SIAM J. Matrix Anal. Appl., 42.2, (2021), pp. 449–474.

[2] F. Van Eeghem and L. De Lathauwer, *Tensor similarity in chemometrics*, in *Comprehensive Chemometrics: Chemical and Biochemical Data Analysis*, (S. Brown, R. Tauler, and B. Walczak, eds.), Elsevier, 2020.

[3] F. Van Eeghem, O. Debals, and L. De Lathauwer, *Tensor similarity in two modes*, IEEE Trans. Signal Process., 66.5, (2018), pp. 1273–1285.