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Matrix-oriented FEM formulation for stationary and time-dependent PDEs on x-normal domains

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The usual spatial finite element discretisation, of arbitrary order $k \in \mathbb{N}$, of elliptic and parabolic partial differential equations takes the form of a linear system or a system of ODEs in the parabolic case. On the so-called *x-normal domains*, we show that the method allows for a Matrix-Oriented formulation, called MO-FEM, [1]. In the elliptic case, the discrete problem takes the form of a *multiterm Sylvester equation*, in the parabolic case a sequence of multiterm Sylvester equations after time discretisation. The proposed framework encompasses the special case $k = 1$ on square and rectangular domains [2], where the discrete problem is a standard (two-term) Sylvester equation.

On square domains, each Sylvester equation can be solved very efficiently with the so-called *reduced method* in the spectral space. On general x-normal domains, when the reduced approach does not apply, we solve each multiterm Sylvester equation apply through the matrix-oriented form of the Preconditioned Conjugate Gradient (MO-PCG) with an ad-hoc preconditioner. The MO-PCG proves more efficient, in terms of computational time and memory occupation, than its standard counterpart in vector form and than MATLAB's built-in direct solver.

As an application, we consider reaction-diffusion PDE systems, where the coupling between diffusion and nonlinear kinetics can lead to the so-called Turing instability. To capture the morphological peculiarities of the Turing patterns, a very fine space discretisation is required, limiting the use of standard (vector-based) ODE solvers in time because of excessive computational costs. To show the advantages of the MO-FEM-PCG to approximate Turing patterns with high spatial resolution, we apply the MO-FEM to a two-species reaction-diffusion system for battery modeling [3].

References

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