Geometry and Arithmetic of Moduli Spaces

Monday, 18 July 2022 - Friday, 22 July 2022

Department of Mathematics, University of Pisa

Book of Abstracts

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Moduli spaces and their uniformizations - 1

Author: Fabrizio Andreatta[1]

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During the first lesson, I will introduce the moduli space of principally polarized abelian varieties. Over the complex numbers, it admits a uniformization via a hermitian symmetric space – the Siegel upper half-space. This admits an embedding into its compact dual, providing an important tool to study automorphic vector bundles and modular forms. In the other two lessons, after introducing this classical setting, I will then outline the p-adic analogue due to the work of P. Scholze.

References:
CL Chai - Siegel moduli schemes and their compactifications over $\mathbb{C}$ - Arithmetic geometry, 1986 - Springer
P Scholze - On torsion in the cohomology of locally symmetric varieties - Annals of Mathematics, 2015 (only section 3)

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Character varieties and representation theory - 1

Author: Emmanuel Letellier[1]

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Given a finite group G, one can define two natural rings: the character ring of G and the center of the group algebra of G. When G is abelian, the two rings are isomorphic via a Fourier transform. In the non-abelian case such a Fourier transform does not exist. In this lecture I will discuss the case where G is the general general linear group over a finite field. We will see how to make a bridge between these two rings through the geometry of character varieties (moduli space of local systems on punctured Riemann sphere), the moduli space of Higgs bundles or quiver varieties.

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Existence of moduli spaces for algebraic stacks - 1

Author: Jarod Alper[1]

[1] *University of Washington*

Corresponding Author: jarod@uw.edu

The aim of this lecture series is to introduce some recent developments in moduli theory which allows for an intrinsic approach to the construction of projective moduli spaces of objects that may have non-finite automorphism groups, e.g. moduli of semistable vector bundles on a curve. We will begin by introducing good moduli spaces for algebraic stacks, which can be viewed as a stack-theoretic categorization of GIT quotients. We will aim to provide necessary and sufficient conditions for the existence of good moduli spaces. This will require introducing the properties of Theta-reductivity and S-completeness, which are valuative criteria requiring the existence of extensions of morphisms over a codimension 2 point. We will motivate these concepts and develop their basic properties

while providing numerous examples. We will then prove that in characteristic 0 that these conditions characterize precisely when an algebraic stack admits a separated good moduli space. In the final lecture, we will explain how this theory can be applied to give an alternative construction of a projective moduli space parameterizing S-equivalence classes of semistable vector bundles.

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Moduli spaces and their uniformizations - 2

Author: Fabrizio Andreatta[1]

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Moduli spaces and their uniformizations - 3

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A Tannakian formalism for Bruhat-Tits buildings

Author: Paul Ziegler[1]

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In the theory of reductive groups over local fields, Bruhat-Tits buildings are the analogues of symmetric spaces in the theory of Lie groups. I will start with an introduction to these objects.

By Goldman-Iwahori, the Bruhat-Tits building of the general linear group GL_n over a local field k can be described as the set of non-archimedean norms on the vector space k^n. I will explain how via a Tannakian formalism this can be generalized to a concrete description of the Bruhat-Tits building of an arbitrary reductive group. This also gives a description of the functor of points of Bruhat-Tits group schemes.

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Character varieties and representation theory - 2

Author: Emmanuel Letellier[1]

[1] *Université Paris Cité*

Corresponding Author: emmanuel.letellier@imj-prg.fr

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An overview of Non-Reductive Geometric Invariant Theory and its applications

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Geometric Invariant Theory (GIT) is a powerful theory for constructing and studying the geometry of moduli spaces in algebraic geometry. In this talk I will give an overview of a recent generalisation of GIT called Non-Reductive GIT, and explain how it can be used to construct and study the geometry of new moduli spaces. These include moduli spaces of unstable objects (for example unstable Higgs/vector bundles), hypersurfaces in weighted projective space, k-jets of curves in $\mathbb{C}^n$ and curve singularities.

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Diophantine problems for Campana pairs - 1

Author: Arne Smeets[1]

[1] *Katholieke Universiteit Leuven*

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I will give an introduction to Diophantine problems for Campana's theory of orbifold pairs, a theory rooted in birational geometry, of which a number of interesting arithmetic applications have emerged during the past few years; the notion of "Campana point" allows one two interpolate between the traditional notions of "rational point" and "integral point" in an interesting way. I will focus on two aspects in particular. In dimension 1, we will look at Mordell-type results for Campana pairs of general type (in particular over function fields). In higher dimensions, we will discuss Manin-type conjectures for Campana pairs of Fano type over number fields.

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Existence of moduli spaces for algebraic stacks - 2

Author: Jarod Alper[1]

[1] *University of Washington*

Corresponding Author: jarod@uw.edu

The aim of this lecture series is to introduce some recent developments in moduli theory which allows for an intrinsic approach to the construction of projective moduli spaces of objects that may have non-finite automorphism groups, e.g. moduli of semistable vector bundles on a curve. We will begin by introducing good moduli spaces for algebraic stacks, which can be viewed as a stack-theoretic categorization of GIT quotients. We will aim to provide necessary and sufficient conditions for the existence of good moduli spaces. This will require introducing the properties of Theta-reductivity and S-completeness, which are valuative criteria requiring the existence of extensions of morphisms over a codimension 2 point. We will motivate these concepts and develop their basic properties while providing numerous examples. We will then prove that in characteristic 0 that these conditions characterize precisely when an algebraic stack admits a separated good moduli space. In the final lecture, we will explain how this theory can be applied to give an alternative construction of a projective moduli space parameterizing S-equivalence classes of semistable vector bundles.

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Diophantine problems for Campana pairs - 2

Author: Arne Smeets[1]

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Non special varieties and potential density

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We discuss the problem of finding a geometric characterization of varieties over number fields where the rational points are potentially dense (i.e. dense after a finite extension of the base field). In the spirit of Lang, there are two necessary conditions: there are no dominant maps to varieties of general type and no étale cover dominates a variety of general type. In joint work with E. Rousseau and J. Wang, we construct examples that show that such conditions are not sufficient for the function field and the hyperbolic analogues, thus giving evidence towards Campana's conjecture.

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Existence of moduli spaces for algebraic stacks - 3

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Drinfeld modular forms

Author: Maria Valentino[1]

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Drinfeld modular forms are the function field counterpart of classical (complex) modular forms. Even if basic theory and definitions resemble that of classical theory there are very deep differences. Relevant ones concern diagonalizability problems, newforms/oldforms definitions and the connection between Hecke eigenvalues and Fourier coefficients. The aim of this talk is to present, after a brief introduction to the Drinfeld world, some new and recent results about these topics.

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Diophantine problems for Campana pairs - 3

Author: Arne Smeets[1]

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I will give an introduction to Diophantine problems for Campana's theory of orbifold pairs, a theory rooted in birational geometry, of which a number of interesting arithmetic applications have emerged during the past few years; the notion of "Campana point" allows one two interpolate between the traditional notions of "rational point" and "integral point" in an interesting way. I will focus on two aspects in particular. In dimension 1, we will look at Mordell-type results for Campana pairs of general type (in particular over function fields). In higher dimensions, we will discuss Manin-type conjectures for Campana pairs of Fano type over number fields.

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Hida Theory for Drinfeld Modular Forms

Author: Marc-Hubert Nicole[1]

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Hida theory describes the p-adic variation of classical modular forms. We shall explain how half of Hida theory neatly extends to Drinfeld modular forms associated to Drinfeld modular varieties of any dimension. Joint work with G. Rosso (Montréal).

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Character varieties and representation theory - 3

Author: Emmanuel Letellier[1]

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Non-reductive good moduli spaces

Author: David Rydh[1]

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I will present a theory of non-reductive good moduli spaces (NRGMS) generalizing the non-reductive geometric invariant theory (NRGIT) introduced by Bérczi, Doran, Hawes, and Kirwan. In particular, the basic results of NRGIT hold for NRGMS and we obtain a framework for affine and projective NRGIT. Even though we are in a non-reductive situation, reductive local structure theorems play a significant role.

I will also introduce topological moduli spaces (TMS) and "positive" non-reductive good moduli spaces (N+GMS) which is a more restrictive notion but also contains the theory of BDHK. For N+GMS there are very powerful local structure theorems and both GMS and N+GMS can be characterized among the TMS. Finally, there is a conjectural existence theorem for N+GMS generalizing the existence theorem of Alper, Halpern-Leistner, and Heinloth.