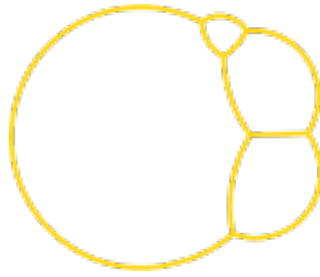


# Isoperimetric Problems



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## Estimates on the Cheeger constant

Monday, 20 June 2022 17:00 (50 minutes)

Given a set  $\Omega \subset \mathbb{R}^N$ , the Cheeger constant is a purely geometrical quantity defined as the infimum 
$$h(\Omega) := \inf \left\{ \frac{P(E)}{|E|} : E \subset \Omega, |E| > 0 \right\}.$$

Despite seeming unassuming, it pops up in many contexts that apparently have nothing in common. To name a few, under some mild regularity assumptions on  $\Omega$ : bounds on the first Dirichlet eigenvalue of the  $p$ -Laplacian; existence of sets in  $\Omega$  or of graphs over  $\Omega$  with prescribed curvature; threshold of vertical load that a flat membrane can sustain before breaking; image reconstruction and denoising. The constant of the unit square has even been a tool in an elementary proof of the Prime Number Theorem!

Given the numerous applications, it is important being able to explicitly compute the constant. This is in general a hard task: a telltale sign is that we do not know the exact value of the constant of the unit cube in dimension  $N \geq 3$ . The computation is (theoretically) feasible for a large class of Jordan domains in the plane [LNS] or in very special cases in general dimension.

If unable to compute the constant, it would be at least desirable to obtain bounds on it: in [LNS] we proved bounds via interior approximations of the set for 2d domains on which, at least on a theoretical level, the constant can be found by solving an algebraic equation; in [JS] a quantitative inequality for the Cheeger constant has been proved in terms of the Riesz asymmetry; in [BPS] bounds of the constant for cylindrical domains  $\Omega = \omega \times [0, L]$  have been shown in terms of the constant of the cross-section  $\omega$ .

[BPS] G. Buttazzo, A. Pratelli, and G. Saracco. Upper and lower bounds on the first Dirichlet eigenvalue of the  $p$ -Laplacian in cylindrical domains, and existence of minimizers of a shape optimization problem. Forthcoming.

[JS] V. Julin and G. Saracco. “Quantitative lower bounds to the Euclidean and the Gaussian Cheeger constants.” In: *Ann. Fenn. Math.* 46.2 (2021), pp. 1071–1087.

[LNS] G. P. Leonardi, R. Neumayer, and G. Saracco. “The Cheeger constant of a Jordan domain without necks.” In: *Calc. Var. Partial Differential Equations* 56.6 (2017), p. 164.

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