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Isoperimetric Multi-Bubble Problems - Old and New

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The classical isoperimetric inequality in Euclidean space $\mathbb{R}^n$ states that among all sets ("bubbles") of prescribed volume, the Euclidean ball minimizes surface area. One may similarly consider isoperimetric problems for more general metric-measure spaces, such as on the sphere $\mathbb{S}^n$ and on Gauss space $\mathbb{G}^n$. Furthermore, one may consider the "multi-bubble" partitioning problem, where one partitions the space into $q \geq 2$ (possibly disconnected) bubbles, so that their total common surface-area is minimal. The classical case, referred to as the single-bubble isoperimetric problem, corresponds to $q = 2$; the case $q = 3$ is called the double-bubble problem, and so on.

In 2000, Hutchings, Morgan, Ritoré and Ros resolved the Double-Bubble conjecture in Euclidean space $\mathbb{R}^3$ (and this was subsequently resolved in $\mathbb{R}^n$ as well) – the optimal partition into two bubbles of prescribed finite volumes (and an exterior unbounded third bubble) which minimizes the total surface-area is given by three spherical caps, meeting at 120°-degree angles. A more general conjecture of J.~Sullivan from the 1990's asserts that when $q \leq n + 2$, the optimal multi-bubble partition of $\mathbb{R}^n$ (as well as $\mathbb{S}^n$) is obtained by taking the Voronoi cells of q equidistant points in $\mathbb{S}^n$ and applying appropriate stereographic projections to $\mathbb{R}^n$ (and backwards).

In 2018, together with Joe Neeman, we resolved the analogous multi-bubble conjecture on the optimal partition of Gauss space $\mathbb{G}^n$ into $q \leq n + 1$ bubbles – the unique optimal partition is given by the Voronoi cells of (appropriately translated) q equidistant points. In this talk, we will describe our approach in that work, as well as recent progress on the multi-bubble problem on $\mathbb{R}^n$ and $\mathbb{S}^n$. In particular, we show that minimizing partitions are always spherical when $q \leq n+1$, and we resolve the latter conjectures when in addition $q \leq 6$ (e.g. the triple-bubble conjecture in $\mathbb{R}^3$ and $\mathbb{S}^3$, and the quadruple-bubble conjecture in $\mathbb{R}^4$ and $\mathbb{S}^4$).

Based on joint work (in progress) with Joe Neeman

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