New Perspectives on Hyperkähler Manifolds - A Celebration of Dimitri Markushevich's (60+2)nd Birthday

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BellaVista Relax Hotel

Book of Abstracts

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On algebraically coisotropic submanifolds

Author: Ekaterina Amerik[1]

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This is joint work with F. Campana. Recall that a submanifold X in a holomorphic symplectic manifold M is said to be coisotropic if the corank of the restriction of the holomorphic symplectic form s is maximal possible, that is equal to the codimension of X. In particular a hypersurface is always coisotropic. The kernel of the restriction of s defines a foliation on X; if it is a fibration, X is said to be algebraically coisotropic. A few years ago we proved that a non-uniruled algebraically coisotropic hypersurface $X \subset M$ is a finite etale quotient of $C \times Y \subset S \times Y$, where $C \subset S$ is a curve in a holomorphic symplectic surface, and Y is arbitrary holomorphic symplectic. We prove some partial results on the higher-codimensional analogue of this, with emphasis on the abelian case.

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Prym fibrations as irreducible symplectic varieties

Author: Chiara Camere[1]

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In this talk, I will first recall the construction of Lagrangian fibrations by Prym varieties starting from a K3 surface with a non-symplectic involution. Then I will discuss a criterion to ensure that the normalizazion of such a fibration is an irreducible symplectic variety. This is joint work in progress with E. Brakkee, A. Grossi, L. Pertusi, G. Saccà, and A. Viktorova.

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Instanton sheaves of low charge on Fano threefolds

Author: Gaia Comaschi[1]

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Let X be a Fano threefold of Picard number one and of index i_X . A rank 2 instanton sheaf of charge k on X is defined as a μ-semistable rank 2 torsion-free sheaf F having Chern classes $c_1(F) = -r_X$, $c_2(F) = k$, $c_3(F) = 0$, with $r_X \in \{0, 1\}$ such that $r_X \equiv i_X \bmod 2$. Locally free instantons, originally defined on the projective space and later generalised on other Fano threefolds X, had been largely studied by several authors in the past years; their moduli spaces present an extremely rich geometry and useful applications to the study of curves on X. In this talk, I will illustrate several features of non-locally free instantons of low charge on 3-dimensional quadrics and cubics. I will focus in particular on the role that they play in the study of the Gieseker-Maruyama moduli space $M_X(2; -h, k, 0)$ and describe how we can still relate these sheaves to curves on X.

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On the even-odd instanton correspondence for Fano threefolds

Author: Daniele Faenzi[1]

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Instanton bundles carry rich geometric data of a Fano threefold X. For instance, for low charge their moduli spaces are related to the fibres of the period map: this was shown by A. Iliev and D. Markushevich for threefolds of genus 8 – we will review ongoing work with A. Verra for the case of genus 10.

Depending on the parity of the index of X, these bundles exhibit different behaviour, yet their moduli spaces share some common features which I will review, starting with the monadic description.

A conjecture of A. Kuznetsov implies that, for some specific charges, there should be a correspondence between even and odd instantons moduli inducing a birational transformation of the associated projective bundles over threefolds having equivalent Kuznetsov categories. I will discuss work in progress with S. Zhang and S. Feyzbakhsh about this conjecture for genus 10 and 12.

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Motives of moduli spaces and hyper-Kähler varieties

Author: Salvatore Floccari[1]

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The best-known examples of hyper-Kähler varieties are obtained considering moduli spaces of sheaves on a K3 or abelian surface. The motive of a hyper-Kähler variety so constructed should be entirely governed by the underlying surface; more concretely, we expect to be able to give a formula for such a motive in terms of that of the surface. I will explain what is known about this problem and some related conjectures. I will then present a formula for the motive of O'Grady's six dimensional varieties obtained via the desingularization of certain moduli spaces of sheaves on an abelian surface, and discuss some consequences.

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Wirthmüller's theorem and modular differential equations for basic Jacobi forms

Author: Valery Gritsenko[1]

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The graded rings of Weyl-invariant weak Jacobi forms play an important role in the theory of Frobenius varieties.
Basis generators of these rings are also generating functions of some quantities (indices of vector bundles, Gromov-Witten invariants, multiplicities of positive roots of Lorentzian Kac-Moody algebras). The case of the root system D_n is very interesting from this point of view.
With my formal Ph.D. student Dimitri Adler, we generalise the idea of the D_8-tower of Jacobi forms coming for the theory of reflective modular form, and give a simple constructive proof of the Wirthmüller theorem for the root system D_n. Then we study the modular differential equations for the most important generators. A surprise is that there are unusual anomalies for some n and reasonable differences for varieties such as $K3$ surfaces and CY_3.

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IHS varieties as symplectic quotients of IHS manifolds

Author: Annalisa Grossi[1]

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We aim at constructing IHS varieties with a trivial algebraic regular fundamental group starting from an IHS manifold X and a finite group G of symplectic actions on X. I will present some results in the case where X is an IHS fourfold of generalized Kummer type and the action is induced from the abelian surface, and I will present some work in progress in the case of symplectic automorphisms of Fano varieties of lines on a cubic 4-fold induced by automorphisms of the cubic. This is a joint work in pregress with Bertini, Capasso, Mauri and Mazzon.

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Walls and asymptotics for Bridgeland stability conditions on threefolds

Author: Marcos Jardim[1]

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Let X be a smooth projective threefold of Picard number one for which the generalized Bogomolov–Gieseker inequality holds, and consider the geometric Brigdeland stability conditions conjectured by Macri–Bayer–Toda. We characterize limit semistable objects, showing that these are Gieseker semistable sheaves for large values of $|\beta|$ and a higher rank generalization of PT stable pairs for large values of α. We also discuss properties of walls and provide a precise description of the Bridgeland moduli spaces for Chern characters of the form $(r, 0, d, 0)$ in certain regions of the (α, β) plane.

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From quartic double solids to Gushel-Mukai threefolds through derived categories

Quartic double solids and Gushel-Mukai threefolds are examples of prime Fano threefolds known to share many numerical properties. In the talk, I will explain how one can construct a smooth and proper over the base family of triangulated categories with special fiber the nontrivial component of the derived category of a quartic double solid and general fiber the nontrivial component of the derived category of a Gushel-Mukai threefold. As a consequence one obtains smooth families of Fano surfaces and intermediate Jacobians of these threefolds. This is a joint work in progress with Evgeny Shinder.

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Fano fourfolds of K3 type

Author: Laurent Manivel[1]

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A joint project with M. Bernardara, E. Fatighenti and F. Tanturri aims at classifying all Fano fourfolds that can be obtained as zero loci of sections of vector bundles on Grassmannians, or more generally products of flag manifolds. Among those, we found 64 families of Fano fourfold of K3 type, whose Hodge structures look like that of a K3 surface. Such varieties usually exhibit some interesting geometry, potentially related to hyperKähler varieties. I will present and discuss some of the most interesting ones.

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Rational Hodge isometries of hyper-Kahler varieties of $K3[n]$-type are algebraic

Author: Eyal Markman[1]

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Let X and Y be compact hyper-Kahler manifolds deformation equivalence to the Hilbert scheme of length n subschemes of a $K3$ surface. A cohomology class in their product $X \times Y$ is an analytic correspondence, if it belongs to the subring generated by Chern classes of coherent analytic sheaves. Let f be a Hodge isometry of their second rational cohomologies with respect to the Beauville-Bogomolov-Fujiki pairings. We prove that f is induced by an analytic correspondence. We furthermore lift f to an analytic correspondence F between their total rational cohomologies, which is a Hodge isometry with respect to the Mukai pairings, and which preserves the gradings up to sign. When X and Y are projective the correspondences f and F are algebraic.

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Non-archimedean geometry and degenerations of CY hypersurfaces

Author: Enrica Mazzon[1]

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SYZ predicts that Calabi-Yau varieties admit special Lagrangian fibrations which conjecturally should be determined by non-archimedean data. With Léonard Pille-Schneider, I constructed a non-archimedean analogue of the classical SYZ fibration for quartic K3 surfaces and quintic 3-folds, which is compatible with expectations from mirror symmetry. In a work in progress with Mattias Jonsson and Nick McCleerey, we solve a non-archimedean conjecture which is the missing step to prove that classical SYZ fibrations exist on a large open region of CY hypersurfaces in $\mathbb{P}^n$.

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Construction of compact hyperkähler orbifolds

Author: Grégoire Menet[1]

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The hyperkähler orbifolds can be seen as one of the simplest generalization of the hyperkähler manifolds in the singular setting. The general theory can be generalized quite easily in this orbifold

framework due to the existence of twistor spaces. In this talk, we will see how to construct examples of hyperkähler orbifolds. In particular, we will provide more than 30 different deformation classes of these orbifolds in dimension 4.

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Irrational Gushel Mukai threefolds

Author: Giovanni Mongardi[1]

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We construct an explicit complex smooth Fano threefold with Picard number 1, index 1, and degree 10 (a Gushel-Mukai threefold) and prove that it is not rational by showing that its intermediate Jacobian has a faithful $\mathsf{PSL}(2, F11)$ action. The construction is based on a very special double EPW sextic. This is joint work with O. Debarre.

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Moduli of complexes on a Veronese double cone

Author: Franco Rota[1]

[1] *University of Glasgow, UK*

I will describe moduli spaces of complexes in the derived category of a general Veronese double cone. The focus will be on objects of the same class of ideal sheaves of lines: first, one observes that both the space of Gieseker stable sheaves and that of complexes in the Kuznetsov component admit two components. One component parametrizes ideal sheaves of lines and appears in both moduli spaces, the appearance of the additional ones is a behavior special to low degree. We show that the additional components are not directly related by a wall-crossing, by describing an intermediate moduli space as a space of Pandharipande-Thomas stable pairs. This is joint work with Marin Petkovic.

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Moduli spaces as Irreducible Symplectic Varieties

Author: Giulia Saccà[1]

[1] *Columbia University, USA*

Recent developments by Druel, Greb-Guenancia-Kebekus, Horing-Peternell have led to the formulation of a decomposition theorem for singular (klt) projective varieties with numerical trivial canonical class. Irreducible symplectic varieties are one of the building blocks provided by this theorem, and the singular analogue of irreducible hyper-Kahler manifolds. In this talk I will show that moduli spaces of Bridgeland stable objects on the Kuznetsov component of a cubic fourfold with respect to a generic stability condition are always projective irreducible symplectic varieties. This builds on the recent work of Bayer-Lahoz-Macri-Neuer-Perry-Stellari, which, ending a long series of results by several authors, proved the analogue statement in the smooth case.

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Moduli with Dima

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Complex Reflection Groups, K3 surfaces and Lehrer-Springer theory

Author: Alessandra Sarti[1]

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I will talk about a long project in collaboration with C. Bonnafé, that relates K3 surfaces and complex reflection groups. This generalizes and explains some results of 2003 by W. Barth and myself. In fact by using complex reflection groups and Lehrer-Springer theory we obtained the following three main results:

1. Classification of some finite groups of maximum order acting on K3 surfaces.
2. Classification of all K3 surfaces that one can obtain as quotient of surfaces in complex projective three space by certain subgroups of finite complex reflection groups of rank four.
3. Description of elliptic fibrations on the previous K3 surfaces.

After an overview of the results, I will explain more in detail point 2., in particular I will introduce some Lehrer-Springer theory from the theory of complex reflection groups, which is a fundamental tool to avoid a case-by-case analysis in the classification.

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Lagrangian fibrations by Prym varieties and their dual fibrations

Author: Justin Sawon[1]

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Lagrangian fibrations are fibrations of hyperkahler manifolds and orbifolds by abelian varieties. Fibrations by Prym varieties were constructed by Markushevich-Tikhomirov, Arbarello-Sacca-Ferretti, and Matteini. The 'spectral curves' of the Markushevich-Tikhmirov and Matteini systems lie in K3 double covers of del Pezzo surfaces of degree two and three, respectively.
In this talk, we consider a Prym fibration in dimension six obtained from spectral curves in a K3 double cover of a degree one del Pezzo. We construct its dual Lagrangian fibration by imitating ideas of Menet, using Pantazis's construction of dual Prym varieties. We speculate on the relation of this (new?) Lagrangian fibration to the Matteini system. This is joint work with Chen Shen.

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Moduli components of semistable rank two sheaves with odd determinant and small charge on projective space

Author: Alexander S. Tikhomirov[1]

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We study the moduli space of semistable rank two semistable torsion-free sheaves with Chern classes $(c_1, c_2, c_3) = (-1, 2n, m)$ on the projective 3-space. We give a complete description of all irreducible components of this moduli space in the case $(c_1, c_2, c_3) = (-1, 2, m)$ for non-negative values for m; all components turn out to be rational. We also prove that this moduli space is connected. This is a joint work with Ch. Almeida and M. Jardim.